Pls state all the theorems that you are using irrespective of how simple it might be
The time limit for this paper is 3 hrs.
You are not allowed any documents or sheets with you and this is a totally closed book examination.
Each question carries 50 points
Best of luck !!!

Full marks that can be earned is 200

1. State and prove Weistrass's approximation theorem

2. State the theorem for the error of linear interpolation of a C^n function. Hence prove it

3. Prove that the best approximation out of a finite dimensional subspace Y of the real vector space $C[a, b]$ is unique iff Y satisfies the Haar condition.

4. Let Ω be an open subspace of R^2. Let K be a triangle $\subset \Omega$ Let the approximations of the function is done using linear ($P1$) elements. Assume that triangle K has vertices $(0, 0), (h, 0), (0, h)$ numbered 1,2,3. Then show that

$$a_{ij}^K = \int_k \bigtriangledown \phi_i \bigtriangledown \phi_j dx$$

using P1 approximation on the triangle K is

$$\begin{pmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0 & 0.5 \end{pmatrix}$$